Differentiation - Concavity

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Questions in past papers often come up combined with other topics.
Topic tags have been given for each question to enable you to know if you can do the question or whether you need to wait to cover the additional topic(s).

Scan the QR code(s) or click the link for instant detailed model solutions!

Question 1

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Applications of Integration, Differentiation

Subtopics: Vertical Tangents, Local or Relative Minima and Maxima, Concavity, Tangents To Curves, Differentiation Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2001 / Difficulty: Somewhat Challenging / Question Number: 4

4. Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by $h'(x) = \frac{x^2 - 2}{x}$ for all $x \neq 0$.

(a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(b) On what intervals, if any, is the graph of h concave up? Justify your answer.

(c) Write an equation for the line tangent to the graph of h at $x = 4$.

(d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

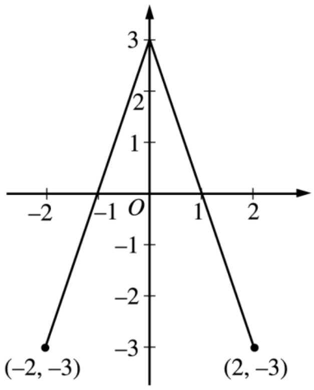
Question 2

Qualification: AP Calculus AB

Areas: Differentiation, Integration, Applications of Differentiation

Subtopics: Fundamental Theorem of Calculus (Second), Integration Technique – Geometric Areas, Derivative Graphs, Integration - Area Under A Curve, Increasing/Decreasing , Concavity, Integration Graphs

Paper: Part B-Non-Calc / Series: 2002 / Difficulty: Hard / Question Number: 4



Graph of f

4. The graph of the function f shown above consists of two line segments. Let g be the function given by $g(x) = \int_0^x f(t)\,dt$.

(a) Find $g(-1)$, $g'(-1)$, and $g''(-1)$.

(b) For what values of x in the open interval $(-2, 2)$ is g increasing? Explain your reasoning.

(c) For what values of x in the open interval $(-2, 2)$ is the graph of g concave down? Explain your reasoning.

(d) On the axes provided, sketch the graph of g on the closed interval $[-2, 2]$.
(Note: The axes are provided in the pink test booklet only.)

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Question 3

Qualification: AP Calculus AB

Areas: Integration, Applications of Differentiation, Limits and Continuity

Subtopics: Properties of Integrals, Fundamental Theorem of Calculus (First), Concavity, Tangents To Curves, Mean Value Theorem, Continuities and Discontinuities, Derivative Tables

Paper: Part B-Non-Calc / Series: 2002 / Difficulty: Hard / Question Number: 6

x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

6. Let f be a function that is differentiable for all real numbers. The table above gives the values of f and its derivative f' for selected points x in the closed interval $-1.5 \le x \le 1.5$. The second derivative of f has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

(a) Evaluate $\int_0^{1.5} (3f'(x) + 4)\, dx$. Show the work that leads to your answer.

(b) Write an equation of the line tangent to the graph of f at the point where $x = 1$. Use this line to approximate the value of $f(1.2)$. Is this approximation greater than or less than the actual value of $f(1.2)$? Give a reason for your answer.

(c) Find a positive real number r having the property that there must exist a value c with $0 < c < 0.5$ and $f''(c) = r$. Give a reason for your answer.

(d) Let g be the function given by $g(x) = \begin{cases} 2x^2 - x - 7 & \text{for } x < 0 \\ 2x^2 + x - 7 & \text{for } x \ge 0 . \end{cases}$

The graph of g passes through each of the points $(x, f(x))$ given in the table above. Is it possible that f and g are the same function? Give a reason for your answer.

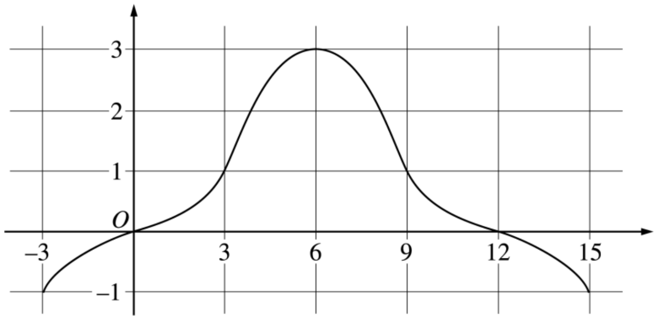
Question 4

Qualification: AP Calculus AB

Areas: Integration, Applications of Integration, Applications of Differentiation, Differentiation

Subtopics: Fundamental Theorem of Calculus (Second), Derivative Graphs, Concavity, Increasing/Decreasing , Riemann Sums – Trapezoidal Rule

Paper: Part B-Non-Calc / Series: 2002-Form-B / Difficulty: Medium / Question Number: 4



Graph of f

4. The graph of a differentiable function f on the closed interval $[-3, 15]$ is shown in the figure above. The graph of f has a horizontal tangent line at $x = 6$. Let $g(x) = 5 + \int_6^x f(t)\,dt$ for $-3 \le x \le 15$.

(a) Find $g(6)$, $g'(6)$, and $g''(6)$.

(b) On what intervals is g decreasing? Justify your answer.

(c) On what intervals is the graph of g concave down? Justify your answer.

(d) Find a trapezoidal approximation of $\int_{-3}^{15} f(t)\,dt$ using six subintervals of length $\Delta t = 3$.

Question 5

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Applications of Integration

Subtopics: Accumulation of Change, Total Amount, Increasing/Decreasing , Concavity, Global or Absolute Minima and Maxima

Paper: Part A-Calc / Series: 2004-Form-B / Difficulty: Hard / Question Number: 2

2. For $0 \leq t \leq 31$, the rate of change of the number of mosquitoes on Tropical Island at time t days is modeled by $R(t) = 5\sqrt{t}\cos\left(\frac{t}{5}\right)$ mosquitoes per day. There are 1000 mosquitoes on Tropical Island at time $t = 0$.

(a) Show that the number of mosquitoes is increasing at time $t = 6$.

(b) At time $t = 6$, is the number of mosquitoes increasing at an increasing rate, or is the number of mosquitoes increasing at a decreasing rate? Give a reason for your answer.

(c) According to the model, how many mosquitoes will be on the island at time $t = 31$? Round your answer to the nearest whole number.

(d) To the nearest whole number, what is the maximum number of mosquitoes for $0 \leq t \leq 31$? Show the analysis that leads to your conclusion.

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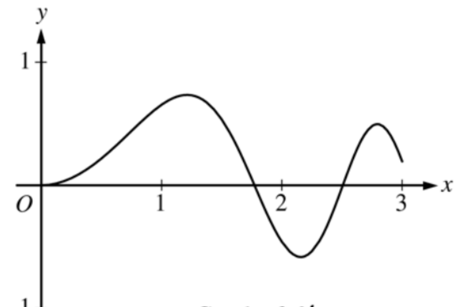
Question 6

Qualification: AP Calculus AB

Areas: Applications of Differentiation

Subtopics: Derivative Graphs, Concavity, Global or Absolute Minima and Maxima, Tangents To Curves

Paper: Part A-Calc / Series: 2006-Form-B / Difficulty: Medium / Question Number: 2



Graph of f'

2. Let f be the function defined for $x \geq 0$ with $f(0) = 5$ and f', the first derivative of f, given by $f'(x) = e^{(-x/4)} \sin(x^2)$. The graph of $y = f'(x)$ is shown above.

(a) Use the graph of f' to determine whether the graph of f is concave up, concave down, or neither on the interval $1.7 < x < 1.9$. Explain your reasoning.

(b) On the interval $0 \leq x \leq 3$, find the value of x at which f has an absolute maximum. Justify your answer.

(c) Write an equation for the line tangent to the graph of f at $x = 2$.

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Question 7

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration

Subtopics: Tangents To Curves, Concavity, Rates of Change (Instantaneous), Riemann Sums – Right, Interpreting Meaning in Applied Contexts

Paper: Part B-Non-Calc / Series: 2007 / Difficulty: Hard / Question Number: 5

t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.6	0.5

5. The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t, where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table above gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \le t \le 12$. The radius of the balloon is 30 feet when $t = 5$.

(Note: The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)

(a) Estimate the radius of the balloon when $t = 5.4$ using the tangent line approximation at $t = 5$. Is your estimate greater than or less than the true value? Give a reason for your answer.

(b) Find the rate of change of the volume of the balloon with respect to time when $t = 5$. Indicate units of measure.

(c) Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t)\,dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t)\,dt$ in terms of the radius of the balloon.

(d) Is your approximation in part (c) greater than or less than $\int_0^{12} r'(t)\,dt$? Give a reason for your answer.

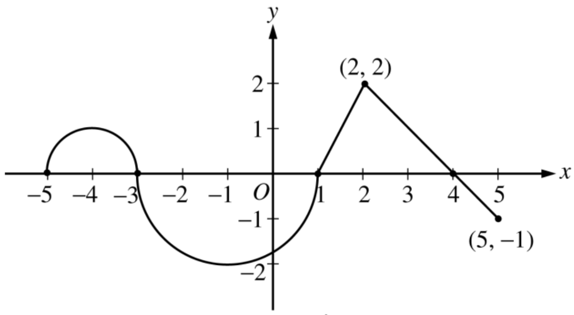
Question 8

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration, Applications of Integration

Subtopics: Integration Technique – Geometric Areas, Local or Relative Minima and Maxima, Points Of Inflection, Concavity, Increasing/Decreasing , Derivative Graphs, Global or Absolute Minima and Maxima

Paper: Part B-Non-Calc / Series: 2007-Form-B / Difficulty: Easy / Question Number: 4



Graph of f'

4. Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.

(a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.

(b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.

(c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.

(d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \le x \le 5$. Explain your reasoning.

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Question 9

Qualification: AP Calculus AB

Areas: Differential Equations, Applications of Differentiation

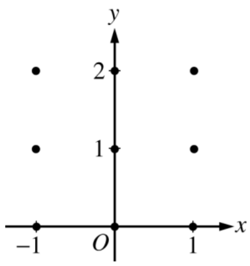
Subtopics: Sketching Slope Field, Concavity, Initial Conditions in Differential Equation, Local or Relative Minima and Maxima, Verifying Solutions to Differential Equation

Paper: Part B-Non-Calc / Series: 2007-Form-B / Difficulty: Somewhat Challenging / Question Number: 5

5. Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}x + y - 1$.

(a) On the axes provided, sketch a slope field for the given differential equation at the nine points indicated.

(Note: Use the axes provided in the exam booklet.)



(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Describe the region in the xy-plane in which all solution curves to the differential equation are concave up.

(c) Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = 1$. Does f have a relative minimum, a relative maximum, or neither at $x = 0$? Justify your answer.

(d) Find the values of the constants m and b, for which $y = mx + b$ is a solution to the differential equation.

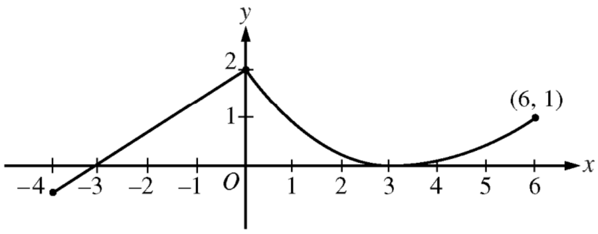
Question 10

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Differentiation

Subtopics: Differentiability, Rates of Change (Average), Mean Value Theorem, Concavity, Fundamental Theorem of Calculus (Second)

Paper: Part A-Calc / Series: 2009-Form-B / Difficulty: Somewhat Challenging / Question Number: 3



Graph of f

3. A continuous function f is defined on the closed interval $-4 \le x \le 6$. The graph of f consists of a line segment and a curve that is tangent to the x-axis at $x = 3$, as shown in the figure above. On the interval $0 < x < 6$, the function f is twice differentiable, with $f''(x) > 0$.

(a) Is f differentiable at $x = 0$? Use the definition of the derivative with one-sided limits to justify your answer.

(b) For how many values of a, $-4 \le a < 6$, is the average rate of change of f on the interval $[a, 6]$ equal to 0 ? Give a reason for your answer.

(c) Is there a value of a, $-4 \le a < 6$, for which the Mean Value Theorem, applied to the interval $[a, 6]$, guarantees a value c, $a < c < 6$, at which $f'(c) = \frac{1}{3}$? Justify your answer.

(d) The function g is defined by $g(x) = \int_0^x f(t)\,dt$ for $-4 \le x \le 6$. On what intervals contained in $[-4, 6]$ is the graph of g concave up? Explain your reasoning.

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Question 11

Qualification: AP Calculus AB

Areas: Differential Equations, Applications of Differentiation

Subtopics: Tangents To Curves, Concavity, Particular Solution of Differential Equation, Initial Conditions in Differential Equation, Separation of Variables in Differential Equation, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2010 / Difficulty: Easy / Question Number: 6

6. Solutions to the differential equation $\frac{dy}{dx} = xy^3$ also satisfy $\frac{d^2y}{dx^2} = y^3\left(1 + 3x^2y^2\right)$. Let $y = f(x)$ be a particular solution to the differential equation $\frac{dy}{dx} = xy^3$ with $f(1) = 2$.

(a) Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$.

(b) Use the tangent line equation from part (a) to approximate $f(1.1)$. Given that $f(x) > 0$ for $1 < x < 1.1$, is the approximation for $f(1.1)$ greater than or less than $f(1.1)$? Explain your reasoning.

(c) Find the particular solution $y = f(x)$ with initial condition $f(1) = 2$.

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Question 12

Qualification: AP Calculus AB

Areas: Applications of Differentiation

Subtopics: Tangents To Curves, Concavity

Paper: Part A-Calc / Series: 2010-Form-B / Difficulty: Easy / Question Number: 2

2. The function g is defined for $x > 0$ with $g(1) = 2$, $g'(x) = \sin\left(x + \frac{1}{x}\right)$, and $g''(x) = \left(1 - \frac{1}{x^2}\right)\cos\left(x + \frac{1}{x}\right)$.

(a) Find all values of x in the interval $0.12 \le x \le 1$ at which the graph of g has a horizontal tangent line.

(b) On what subintervals of $(0.12, 1)$, if any, is the graph of g concave down? Justify your answer.

(c) Write an equation for the line tangent to the graph of g at $x = 0.3$.

(d) Does the line tangent to the graph of g at $x = 0.3$ lie above or below the graph of g for $0.3 < x < 1$? Why?

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Question 13

Qualification: AP Calculus AB

Areas: Differential Equations, Applications of Differentiation

Subtopics: Tangents To Curves, Concavity, Particular Solution of Differential Equation, Initial Conditions in Differential Equation, Separation of Variables in Differential Equation, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2011 / Difficulty: Medium / Question Number: 5

5. At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function W models the total amount of solid waste stored at the landfill. Planners estimate that W will satisfy the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ for the next 20 years. W is measured in tons, and t is measured in years from the start of 2010.

(a) Use the line tangent to the graph of W at $t = 0$ to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time $t = \frac{1}{4}$).

(b) Find $\frac{d^2W}{dt^2}$ in terms of W. Use $\frac{d^2W}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time $t = \frac{1}{4}$.

(c) Find the particular solution $W = W(t)$ to the differential equation $\frac{dW}{dt} = \frac{1}{25}(W - 300)$ with initial condition $W(0) = 1400$.

Question 14

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration

Subtopics: Local or Relative Minima and Maxima, Concavity, Differentiation Technique – Standard Functions, Integration Technique – Standard Functions

Paper: Part B-Non-Calc / Series: 2011-Form-B / Difficulty: Medium / Question Number: 4

4. Consider a differentiable function f having domain all positive real numbers, and for which it is known that $f'(x) = (4 - x)x^{-3}$ for $x > 0$.

   (a) Find the x-coordinate of the critical point of f. Determine whether the point is a relative maximum, a relative minimum, or neither for the function f. Justify your answer.

   (b) Find all intervals on which the graph of f is concave down. Justify your answer.

   (c) Given that $f(1) = 2$, determine the function f.

Question 15

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differential Equations

Subtopics: Derivative Graphs, Concavity, Separation of Variables in Differential Equation, Integration Technique – Standard Functions, Initial Conditions in Differential Equation, Particular Solution of Differential Equation

Paper: Part B-Non-Calc / Series: 2012 / Difficulty: Somewhat Challenging / Question Number: 5

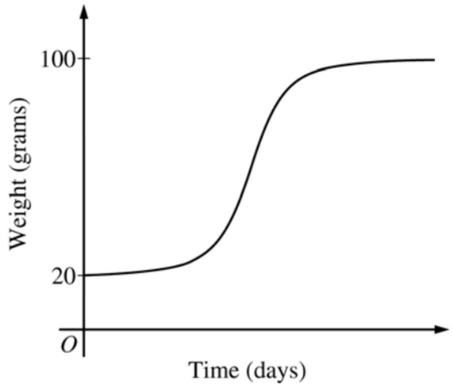
5. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.



(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

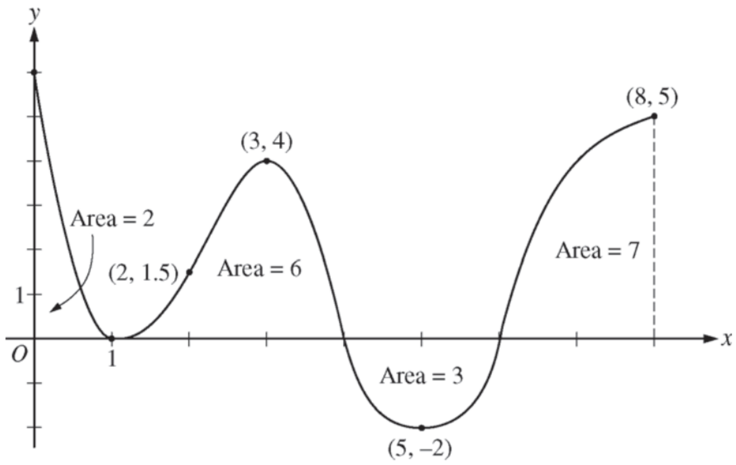
Question 16

Qualification: AP Calculus AB

Areas: Applications of Differentiation

Subtopics: Local or Relative Minima and Maxima, Global or Absolute Minima and Maxima, Concavity, Increasing/Decreasing , Implicit Differentiation, Tangents To Curves, Derivative Graphs

Paper: Part B-Non-Calc / Series: 2013 / Difficulty: Somewhat Challenging / Question Number: 4



Graph of f'

4. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the closed interval $0 \le x \le 8$. The graph of f' has horizontal tangent lines at $x = 1$, $x = 3$, and $x = 5$. The areas of the regions between the graph of f' and the x-axis are labeled in the figure. The function f is defined for all real numbers and satisfies $f(8) = 4$.

(a) Find all values of x on the open interval $0 < x < 8$ for which the function f has a local minimum. Justify your answer.

(b) Determine the absolute minimum value of f on the closed interval $0 \le x \le 8$. Justify your answer.

(c) On what open intervals contained in $0 < x < 8$ is the graph of f both concave down and increasing? Explain your reasoning.

(d) The function g is defined by $g(x) = (f(x))^3$. If $f(3) = -\frac{5}{2}$, find the slope of the line tangent to the graph of g at $x = 3$.

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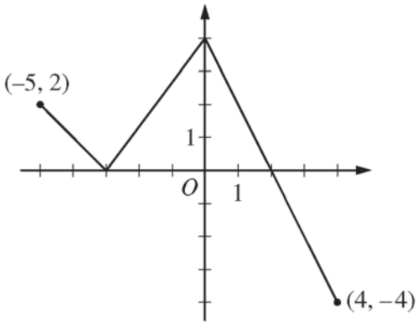
Question 17

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation, Integration

Subtopics: Integration Technique – Geometric Areas, Increasing/Decreasing , Concavity, Differentiation Technique - Quotient Rule, Tangents To Curves, Differentiation Technique – Chain Rule, Integration Graphs

Paper: Part B-Non-Calc / Series: 2014 / Difficulty: Medium / Question Number: 3



Graph of f

3. The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above. Let g be the function defined by $g(x) = \int_{-3}^{x} f(t)\,dt$.

(a) Find $g(3)$.

(b) On what open intervals contained in $-5 < x < 4$ is the graph of g both increasing and concave down? Give a reason for your answer.

(c) The function h is defined by $h(x) = \dfrac{g(x)}{5x}$. Find $h'(3)$.

(d) The function p is defined by $p(x) = f\left(x^2 - x\right)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$.

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Question 18

Qualification: AP Calculus AB

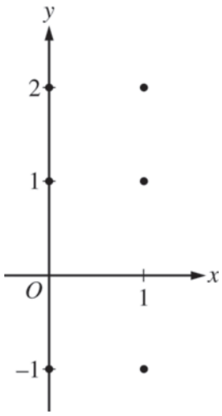
Areas: Differential Equations, Applications of Differentiation

Subtopics: Sketching Slope Field, Concavity, Local or Relative Minima and Maxima, Verifying Solutions to Differential Equation

Paper: Part B-Non-Calc / Series: 2015 / Difficulty: Medium / Question Number: 4

4. Consider the differential equation $\frac{dy}{dx} = 2x - y$.

(a) On the axes provided, sketch a slope field for the given differential equation at the six points indicated.



(b) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Determine the concavity of all solution curves for the given differential equation in Quadrant II. Give a reason for your answer.

(c) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(2) = 3$. Does f have a relative minimum, a relative maximum, or neither at $x = 2$? Justify your answer.

(d) Find the values of the constants m and b for which $y = mx + b$ is a solution to the differential equation.

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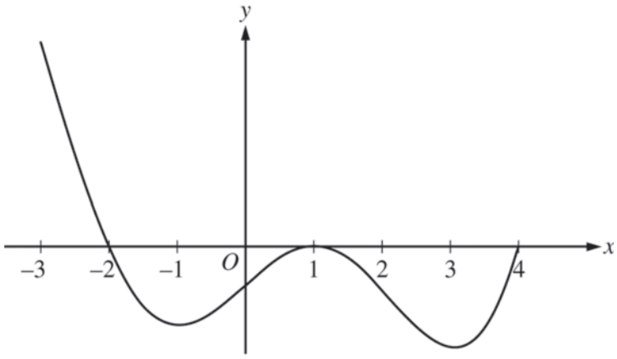
Question 19

Qualification: AP Calculus AB

Areas: Applications of Differentiation

Subtopics: Local or Relative Minima and Maxima, Increasing/Decreasing , Concavity, Points Of Inflection, Derivative Graphs

Paper: Part B-Non-Calc / Series: 2015 / Difficulty: Easy / Question Number: 5



Graph of f'

5. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.

(a) Find all x-coordinates at which f has a relative maximum. Give a reason for your answer.

(b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.

(c) Find the x-coordinates of all points of inflection for the graph of f. Give a reason for your answer.

(d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

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Question 20

Qualification: AP Calculus AB

Areas: Differential Equations, Applications of Differentiation

Subtopics: Tangents To Curves, Concavity, Particular Solution of Differential Equation, Initial Conditions in Differential Equation, Separation of Variables in Differential Equation, Integration Technique - Harder Powers

Paper: Part B-Non-Calc / Series: 2017 / Difficulty: Somewhat Challenging / Question Number: 4

4. At time $t = 0$, a boiled potato is taken from a pot on a stove and left to cool in a kitchen. The internal temperature of the potato is 91 degrees Celsius (°C) at time $t = 0$, and the internal temperature of the potato is greater than 27°C for all times $t > 0$. The internal temperature of the potato at time t minutes can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = -\frac{1}{4}(H - 27)$, where $H(t)$ is measured in degrees Celsius and $H(0) = 91$.

(a) Write an equation for the line tangent to the graph of H at $t = 0$. Use this equation to approximate the internal temperature of the potato at time $t = 3$.

(b) Use $\frac{d^2H}{dt^2}$ to determine whether your answer in part (a) is an underestimate or an overestimate of the internal temperature of the potato at time $t = 3$.

(c) For $t < 10$, an alternate model for the internal temperature of the potato at time t minutes is the function G that satisfies the differential equation $\frac{dG}{dt} = -(G - 27)^{2/3}$, where $G(t)$ is measured in degrees Celsius and $G(0) = 91$. Find an expression for $G(t)$. Based on this model, what is the internal temperature of the potato at time $t = 3$?

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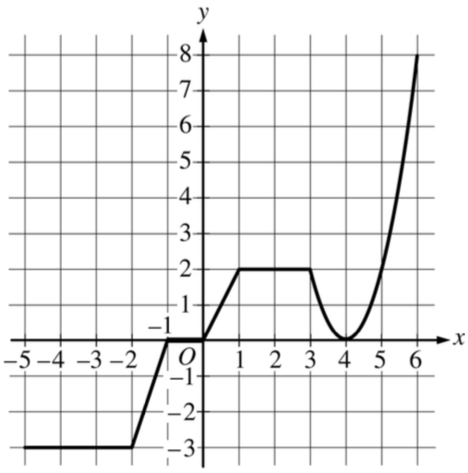
Question 21

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration

Subtopics: Integration Technique – Geometric Areas, Derivative Graphs, Increasing/Decreasing , Concavity, Points Of Inflection, Integration Graphs

Paper: Part B-Non-Calc / Series: 2018 / Difficulty: Medium / Question Number: 3



Graph of g

3. The graph of the continuous function g, the derivative of the function f, is shown above. The function g is piecewise linear for $-5 \le x < 3$, and $g(x) = 2(x-4)^2$ for $3 \le x \le 6$.

(a) If $f(1) = 3$, what is the value of $f(-5)$?

(b) Evaluate $\int_1^6 g(x)\, dx$.

(c) For $-5 < x < 6$, on what open intervals, if any, is the graph of f both increasing and concave up? Give a reason for your answer.

(d) Find the x-coordinate of each point of inflection of the graph of f. Give a reason for your answer.

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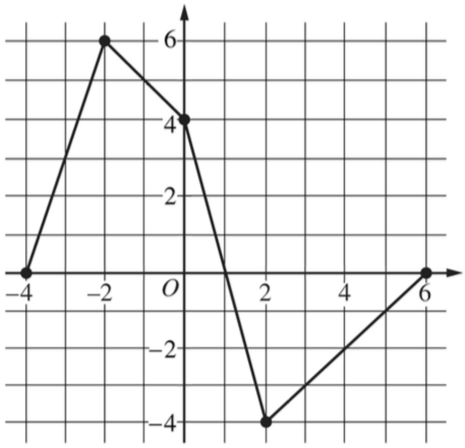
Question 22

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Integration

Subtopics: Concavity, Fundamental Theorem of Calculus (Second), Differentiation Technique – Product Rule, L'Hôpital's Rule, Mean Value Theorem, Rates of Change (Average), Integration Technique – Geometric Areas, Integration Graphs

Paper: Part B-Non-Calc / Series: 2021 / Difficulty: Somewhat Challenging / Question Number: 4



Graph of f

4. Let f be a continuous function defined on the closed interval $-4 \le x \le 6$. The graph of f, consisting of four line segments, is shown above. Let G be the function defined by $G(x) = \int_0^x f(t)\,dt$.

(a) On what open intervals is the graph of G concave up? Give a reason for your answer.

(b) Let P be the function defined by $P(x) = G(x) \cdot f(x)$. Find $P'(3)$.

(c) Find $\lim_{x \to 2} \frac{G(x)}{x^2 - 2x}$.

(d) Find the average rate of change of G on the interval $[-4, 2]$. Does the Mean Value Theorem guarantee a value c, $-4 < c < 2$, for which $G'(c)$ is equal to this average rate of change? Justify your answer.

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Question 23

Qualification: AP Calculus AB

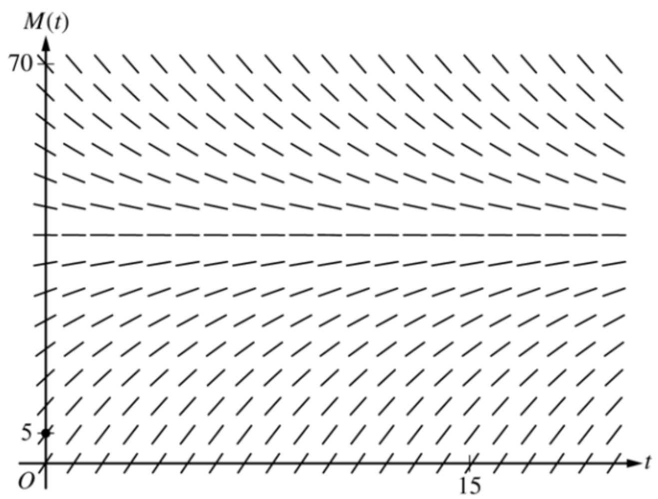
Areas: Differential Equations, Applications of Differentiation

Subtopics: Sketching Slope Field, Tangents To Curves, Concavity, Separation of Variables in Differential Equation, Initial Conditions in Differential Equation, Particular Solution of Differential Equation

Paper: Part B-Non-Calc / Series: 2023 / Difficulty: / Question Number: 3

3. A bottle of milk is taken out of a refrigerator and placed in a pan of hot water to be warmed. The increasing function M models the temperature of the milk at time t, where $M(t)$ is measured in degrees Celsius (°C) and t is the number of minutes since the bottle was placed in the pan. M satisfies the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$. At time $t = 0$, the temperature of the milk is 5°C. It can be shown that $M(t) < 40$ for all values of t.

(a) A slope field for the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ is shown. Sketch the solution curve through the point $(0, 5)$.



(b) Use the line tangent to the graph of M at $t = 0$ to approximate $M(2)$, the temperature of the milk at time $t = 2$ minutes.

(c) Write an expression for $\frac{d^2M}{dt^2}$ in terms of M. Use $\frac{d^2M}{dt^2}$ to determine whether the approximation from part (b) is an underestimate or an overestimate for the actual value of $M(2)$. Give a reason for your answer.

(d) Use separation of variables to find an expression for $M(t)$, the particular solution to the differential equation $\frac{dM}{dt} = \frac{1}{4}(40 - M)$ with initial condition $M(0) = 5$.

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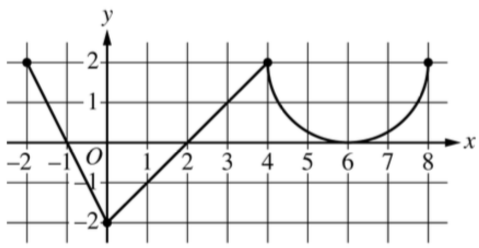
Question 24

Qualification: AP Calculus AB

Areas: Limits and Continuity, Applications of Differentiation

Subtopics: Local or Relative Minima and Maxima, Concavity, Derivative Graphs, Global or Absolute Minima and Maxima, Integration Technique – Geometric Areas, L'Hôpital's Rule, Calculating Limits Algebraically

Paper: Part B-Non-Calc / Series: 2023 / Difficulty: Medium / Question Number: 4



Graph of f'

4. The function f is defined on the closed interval $[-2, 8]$ and satisfies $f(2) = 1$. The graph of f', the derivative of f, consists of two line segments and a semicircle, as shown in the figure.

(a) Does f have a relative minimum, a relative maximum, or neither at $x = 6$? Give a reason for your answer.

(b) On what open intervals, if any, is the graph of f concave down? Give a reason for your answer.

(c) Find the value of $\lim_{x \to 2} \frac{6f(x) - 3x}{x^2 - 5x + 6}$, or show that it does not exist. Justify your answer.

(d) Find the absolute minimum value of f on the closed interval $[-2, 8]$. Justify your answer.

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Question 25

Qualification: AP Calculus AB

Areas: Applications of Differentiation, Differentiation

Subtopics: Derivative Tables, Differentiation Technique – Chain Rule, Concavity, Differentiation Technique – Product Rule, Fundamental Theorem of Calculus (First), Increasing/Decreasing

Paper: Part B-Non-Calc / Series: 2023 / Difficulty: Somewhat Challenging / Question Number: 5

x	0	2	4	7
$f(x)$	10	7	4	5
$f'(x)$	$\frac{3}{2}$	-8	3	6
$g(x)$	1	2	-3	0
$g'(x)$	5	4	2	8

5. The functions f and g are twice differentiable. The table shown gives values of the functions and their first derivatives at selected values of x.

(a) Let h be the function defined by $h(x) = f(g(x))$. Find $h'(7)$. Show the work that leads to your answer.

(b) Let k be a differentiable function such that $k'(x) = (f(x))^2 \cdot g(x)$. Is the graph of k concave up or concave down at the point where $x = 4$? Give a reason for your answer.

(c) Let m be the function defined by $m(x) = 5x^3 + \int_0^x f'(t)\,dt$. Find $m(2)$. Show the work that leads to your answer.

(d) Is the function m defined in part (c) increasing, decreasing, or neither at $x = 2$? Justify your answer.

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